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An Approach Driven Ranking System for Risky Gaits

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A R T I C L E I N F O

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A B S T R A C T

With the increase of fall incidents in emergency cases and expenditures in the healthcare sector, fall prevention has become a very important study. The aim of this paper is to develop a ranking system to capture the risky gaits to aid in the reduction of fall incidents. With this, an approach driven framework under the interval-valued intuitionistic fuzzy environment is developed for decision making. The proposed framework is validated by experimental data and is seen to be effective and reliable. It is applied to risky gaits and a ranking is obtained based on the mindset one chooses. Technically, we also extended the entropy measure to incorporate the hesitancy factor to interval-valued region where a new measuring function is introduced. We believe that the proposed system will provide a better support for rehabilitation arrangements.

1. Introduction

Falls are the most prevalent cause of injuries and it can be fatal. In the literature, many researches related to this issue dealt with the cases where falls have happened. For instance, the inertial-based wearable devices were developed for fall monitoring (Pannurat et al., 2014), a two-stage fall detection system was proposed by Stone and Skubic (2014) and Kau and Chen (2015) studied a smartphone based fall detection and rescue system. Although most of these systems alert the caregivers and prompt for immediate interventions, they only lead to the reduction of fall consequences but do not prevent the injuries. As a result of this, fall prevention systems are pressing a priority. Recently, Rescio et al. (2018) proposed an electromyography based pre-fall detection system that recognizes a fall incident during the initial phase for a better activation of the protection system.

Many studies have been conducted to detect fall incidents after they have occurred but an early prediction of fall risk for support intervention will help in avoiding the painful and costly aftermath that a fall event may cause. Also, fall prevention is of great importance and will have a substantial impact on the well being. Some of the existing fall prevention systems in the healthcare domain is either event based (Ni et al., 2012) or wearable pressure sensor based (Majumder et al., 2013). Most fatal and non-fatal injuries among people are caused by falls. However, there is no data based on real falls as a person would be required to undergo a fall which can be injurious and unsafe (Khan & Hoey, 2017). With the present generations prefer to do everything on the go, young adults have become more prone to falls than never before (Licence et al., 2015). Distracted walking is not only a safety concern but also alters and deteriorates one's gait over a period of time that will increase the risk of falls. Gait parameters are known to be associated with falls (Bautmans et al., 2011) and can assist in screening potential fallers (Senden et al., 2012). Therefore, an analysis of one's gait as early as possible will help to prevent falls and also provide a better health by timely rehabilitation.

The premise of this work is to assess potential fallers risky gait in order to obtain support intervention. Gaits are seen to have a lot of intra-variations for a single person and inter-variations among people. Therefore, in this paper, we propose to employ interval-valued theory with intuitionistic fuzzy sets to handle the inconsistency and spread measures in gait data. The interval-valued theory is necessary as the degrees of belief cannot be mentioned precisely. With crisp values, the inconsistency of human reasoning is difficult to handle. Also, exact information is not feasible all the time as in real life situations, imprecision and vagueness do exist. Decision-makings for healthcare problems is extremely critical and more often than not the data involved in such scenarios revolve around uncertainties.

Intuitionistic fuzzy sets (Atanassov, 1999b) come in handy for representing uncertainties with their characteristic values of membership and non-membership. An extension of intuitionistic fuzzy sets in the interval domain was introduced by Atanassov (1999a) where the interval-valued intuitionistic fuzzy sets (IVIFS) express both the membership and non-membership degrees in the interval forms, but not in the exact values. These are effective tools for researchers when inexact intervals are required to deal with uncertainty (Reiser & Bedregal, 2013). In the literature, multi-criteria decision making techniques and models were also presented by some authors using IVIFS (Hajiagha et al., 2015; Wan & Dong, 2015; Wu & Chiclana, 2012). Having an edge in dealing with

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imprecise data in the real world scenarios, IVIFS has been successfully applied in diverse fields such as warehouse evaluation (Chai et al., 2013), outsourcing providers (Ebrahimnejad et al., 2015), and hotel selection (Cheng, 2018).

In this paper, we propose an approach driven system to rank the risky gaits. To this end, first, medical experts will evaluate the observational temporal and spatial gait characteristics of different subjects. Based on the provided assessments with judgment confidence, the ranking of different subjects are provided in terms of (i) optimistic, (ii) pessimistic, or (iii) neutral mindset approach. This is to help in processing information and provide a better decision with respect to one's mental attitude or belief. A fusion of these three approaches, namely Optimistic Pessimistic Neutral (OPN) approach, is also provided. Beside that, the decision framework has an entropy measure with hesitancy to objectively weight the attributes in different approaches. This will help to tackle the uncertainty in the system. A new measured membership score (MMS) is applied for a more effective ranking of subjects with risky gaits to get support intervention.

The contributions of this paper are, first, we propose a framework that is based on IVIFSs to prevent falls. A geometrical representation of decision making in IVIFS is also presented. Second, a new measuring function is proposed for a more effective ranking. Third, technically, we extend the entropy measure to an interval-valued region. The rest of the paper is as follows. In Section 2, we briefly introduce the IVIFSs, entropy measures, and score functions. In Section 3, we provide the decision-making framework. In Section 4, a geometrical representation of this decision making framework is presented. In Section 5, the framework is applied to a problem of risky gaits and subjects are ranked based on the different approaches. With some final remarks, we conclude in Section 6.

2. Background

In real life, decision making on most of the occasions is generally not binary from the instant we encounter a problem. It is an ambiguous process as human thinking often explores with impreciseness. To incorporate such approximate reasoning where the concept of something neither being absolutely true or false could exist, the fuzzy set theory was introduced and has had various extensions.

2.1. Intuitionistic Fuzzy sets (IFS)

IFS was introduced by Atanassov (1999b) where a set A of the universe of discourse U is given by:

$$A = \{u, \mu_A(u), \nu_A(u) | u \in U\}$$

where μ is the membership degree, $\mu_A : U \rightarrow [0,1]$, ν is the non-membership degree, $\nu_A : U \rightarrow [0,1]$ and

$$0 \leq \mu_A(u) + \nu_A(u) \leq 1, \forall u \in U$$

In this paper, we also adopt the definition proposed in Xu (2007), i.e. if $\alpha_1 = [\mu_1, \nu_2]$ and $\alpha_2 = [\mu_2, \nu_2]$ are two IFSs, and λ is a positive real number:

$$\lambda\alpha = [1 - (1-\mu)^\lambda,\ \nu^\lambda]$$

$$\alpha^\lambda = [\mu^\lambda,\ 1 - (1-\nu)^\lambda]$$

$$\alpha_1 \oplus \alpha_2 = [1 - (1-\mu_1)(1-\mu_2),\ \nu_1\nu_2]$$

$$\alpha_1 \otimes \alpha_2 = [\mu_1\mu_2,\ 1 - (1-\nu_1)(1-\nu_2)]$$

The given α_1 and α_2 are equal if and only if $\mu_1 = \mu_2$ and $\nu_1 = \nu_2$

2.2. Interval valued intuitionistic Fuzzy set (IVIFS)

IVIFS is an extension of IFS Atanassov (1999a) where IVIFS A over the universe of discourse X is of the form

$$A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$$

where μ_A is an interval of membership degrees and ν_A is an interval of non-membership degrees for the element x, both with ranges [0,1]. The lower bounds and upper bounds of μ_A and ν_A are denoted as $\underline{\mu}_A$, $\overline{\mu}_A$, and $\underline{\nu}_A$, $\overline{\nu}_A$ respectively. Therefore, an IVIFS A can be better represented as:

$$A = \{(x, [\underline{\mu}_A(x), \overline{\mu}_A(x)], [\underline{\nu}_A(x), \overline{\nu}_A(x)]) | x \in X\} \tag{1}$$

where the following condition holds true:

$$0 \leq \overline{\mu}_A(x) + \overline{\nu}_A(x) \leq 1 \tag{2}$$

The hesitancy factor, or hesitation degree for an IVIFS A is denoted by h_A where the interval $[\underline{h}_A(x), \overline{h}_A(x)]$ is calculated by:

$$\underline{h}_A = 1 - \overline{\mu}_A - \overline{\nu}_A \tag{3}$$

$$\overline{h}_A = 1 - \underline{\mu}_A - \underline{\nu}_A \tag{4}$$

The hesitancy factor represents the hesitation or the lack of confidence in the element's sureness about something. Decision making often involves uncertainty and IVIFS provides the means to tackle the fuzziness and inexactness associated with it.

2.3. Entropy measure

Uncertainty in a system can be defined by entropy and has been widely used in decision-making problems. An entropy weight model for IVIFSs was proposed by Ye (2010) to determine the criteria weight. Consequently, Wei and Zhang (2015) proposed to solve multi-criteria problems using cosine entropy functions whereas Düğenci (2016) used distance measure based function. Recently, Narayanamoorthy et al. (2019) showcased an entropy based method for industrial applications and Takáč et al. (2018) introduced interval entropy. Among the various measures that exist, some of the existing IVIFS entropy measures are:

(a) Jun Ye's Entropy:

$$E_{JY}(A) = \frac{1}{n}\sum_{i=1}^{n} \frac{1}{\sqrt{2}-1}\Bigg(\cos\frac{\pi(1+\underline{\mu}_A(x_i)+p[\overline{\mu}_A(x_i)-\underline{\mu}_A(x_i)]-\underline{\nu}_A(x_i)-q[\overline{\nu}_A(x_i)-\underline{\nu}_A(x_i)])}{4} + \cos\frac{\pi(1-\underline{\mu}_A(x_i)-p[\overline{\mu}_A(x_i)-\underline{\mu}_A(x_i)]+\underline{\nu}_A(x_i)+q[\overline{\nu}_A(x_i)-\underline{\nu}_A(x_i)])}{4} - 1 \Bigg) \tag{5}$$

This entropy measure was proposed by Ye (2010) to establish an exact model for weighing the criteria and evaluate for the weighted correlation coefficient.

(b) Wei et al. Entropy:

$$E_{WC}(A) = \frac{1}{n}\sum_{i=1}^{n} \frac{\min\{\underline{\mu}_A(x_i), \underline{\nu}_A(x_i)\} + \min\{\overline{\mu}_A(x_i), \overline{\nu}_A(x_i)\} + \underline{h}_A(x_i) + \overline{h}_A(x_i)}{\max\{\underline{\mu}_A(x_i), \underline{\nu}_A(x_i)\} + \max\{\overline{\mu}_A(x_i), \overline{\nu}_A(x_i)\} + \underline{h}_A(x_i) + \overline{h}_A(x_i)} \tag{6}$$

Wei et al. (2011) generalized three existing entropy into one and tried constructing similarity measures.

(c) Liu Jing's Entropy:

$$E_{LJ}(A) = \frac{1}{n}\sum_{i=1}^{n} \frac{2 + \overline{h}_A(x_i) + \underline{h}_A(x_i) - \left|\underline{\mu}_A(x_i) - \underline{\nu}_A(x_i)\right| - \left|\overline{\mu}_A(x_i) - \overline{\nu}_A(x_i)\right|}{2 + \overline{h}_A(x_i) + \underline{h}_A(x_i)} \tag{7}$$

A real-valued continuous function for entropy was introduced by Jing (2013) which increases with respect to intuitionism.

(d) Zhang et al. Entropy:

$$E_{ZX}(A) = 1 - \frac{1}{2n}\sum_{i=1}^{n}\Big[\,|\overline{\mu}_A(x_i) - 0.5| + \big|\underline{\mu}_A(x_i) - 0.5\big| + |\overline{\nu}_A(x_i) - 0.5| + \big|\underline{\nu}_A(x_i) - 0.5\big|\Big] \tag{8}$$

Zhang et al. (2014) presented an entropy measure based on distance along with a new axiomatic definition.

(e) Wei and Zhang's Entropy:

$$E_{WZ}(A) = \frac{1}{n}\sum_{i=1}^{n}\cos\frac{\big|\underline{\mu}_A(x_i) - \underline{\nu}_A(x_i)\big| + |\overline{\mu}_A(x_i) - \overline{\nu}_A(x_i)|}{2\big(2 + \overline{h}_A(x_i) + \underline{h}_A(x_i)\big)}\pi \tag{9}$$

A more sufficient entropy measure for describing uncertain information was proposed by Wei and Zhang (2015).

Here, we consider a few IVIFSs, singletons and non-singletons, to look at the effectiveness of the above-mentioned entropy measures.

Let us consider two IVIFSs $A([0.1,0.2],[0.3,0.4])$ and $B([0.5,0.5],[0.3,0.3])$. It can be intuitively seen that A is fuzzier than B. On using Jun Ye's entropy Eq. (5), we see that $E_{JY}(A) = E_{JY}(B) = 0.96$, making A and B indistinguishable. Also according to Zhang et al. (2014), or the Eq. (8), B is more fuzzy than A as $E_{ZX}(A) = 0.50$ and $E_{ZX}(B) = 0.80$ which is counter-intuitive.

Let the two IVIFSs be $A([0.5,0.5],[0.1,0.1])$ and $B([0.6,0.6],[0.2,0.2])$. We find that $E_{JY}(A) = E_{JY}(B) = 0.83$ and $E_{ZX}(A) = E_{ZX}(B) = 0.60$. A and B stand to have same importance which is irrational. Similar case is observed when considering $A([0.5,0.5],[0.0,0.0])$ and $B([0.7,0.7],[0.2,0.2])$ as $E_{JY}(A) = E_{JY}(B) = 0.74$ and $E_{ZX}(A) = E_{ZX}(B) = 0.50$.

It is observed that the entropy measures produce identical value for two IVIFSs A and B when $|\underline{\mu} - \underline{\nu}|$ and $|\overline{\mu} - \overline{\nu}|$ have the same difference for both. One rationale for this happening is the fact that hesitancy factors of the IVIFSs are not considered during the evaluations of entropy measures.

Entropy measures using Eq. (6), Eq. (7) and Eq. (9) factor in the hesitancy and are seen to produce desirable results for all the above considered IVIFSs, but still have limitations. If the two IVIFSs considered are $A([0.5,0.5],[0.0,0.0])$ and $B([0.6,0.6],[0.2,0.2])$, we see that Eq. (6), Eq. (7) and Eq. (9) failed to differentiate them. They yield $E_{WC}(A) = E_{WC}(B) = 0.50$, $E_{LJ}(A) = E_{LJ}(B) = 0.67$ and $E_{WZ}(A) = E_{WZ}(B) = 0.87$ respectively, producing undesirable results. This urges for the need of a better entropy measure for IVIFS. In Section 3.3 we present an entropy measure which overcomes the above limitations.

2.4. Scoring function

IVIFSs are generally compared and ranked using some form of metric measures. The commonly used score function and accuracy function were proposed by Xu and Jian (2007) and a novel accuracy function for multi-criteria was introduced by Ye (2009). A new uncertainty index for membership and hesitation (Wang et al., 2009) was used to compare two interval-valued intuitionistic fuzzy numbers (IVIFNs), and Nayagam et al. (2011) developed a ranking method. Some of the measure function for an IVIFS α are as follow:

(i) Score function (Xu & Jian, 2007):

$$S_1(\alpha) = \frac{\underline{\mu} + \overline{\mu} - \underline{\nu} - \overline{\nu}}{2} \tag{10}$$

(ii) Accuracy function (Xu & Jian, 2007):

$$S_2(\alpha) = \frac{\underline{\mu} + \overline{\mu} + \underline{\nu} + \overline{\nu}}{2} \tag{11}$$

(iii) Novel Accuracy function (Ye, 2009):

$$S_3(\alpha) = \underline{\mu} + \overline{\mu} - 1 + \frac{\underline{\nu} + \overline{\nu}}{2} \tag{12}$$

(iv) Membership Uncertainty Index function (Wang et al., 2009):

$$S_4(\alpha) = \overline{\mu} + \underline{\nu} - \underline{\mu} - \overline{\nu} \tag{13}$$

(v) New Accuracy function (Nayagam et al., 2011):

$$S_5(\alpha) = \frac{\underline{\mu} + \overline{\mu} + \overline{\nu}(1 - \overline{\mu}) + \underline{\nu}(1 - \underline{\mu})}{2} \tag{14}$$

Let α_1 = ([0.20, 0.20], [0.40, 0.40]), α_2 = ([0.15, 0.25], [0.35, 0.45]), α_3 = ([0.10, 0.30], [0.30, 0.50]), α_4 = ([0.05, 0.35], [0.25, 0.55]). The corresponding values for the IVIFSs using the above mentioned measure functions are:

$S_1(\alpha_1) = S_1(\alpha_2) = S_1(\alpha_3) = S_1(\alpha_4) = -0.20$;
$S_2(\alpha_1) = S_2(\alpha_2) = S_2(\alpha_3) = S_2(\alpha_4) = 0.60$;
$S_3(\alpha_1) = S_3(\alpha_2) = S_3(\alpha_3) = S_3(\alpha_4) = -0.20$;
$S_4(\alpha_1) = S_4(\alpha_2) = S_4(\alpha_3) = S_4(\alpha_4) = 0.00$;
$S_5(\alpha_1) = -0.120$, $S_5(\alpha_2) = -0.118$, $S_5(\alpha_3) = -0.110$, $S_5(\alpha_4) = -0.098$;

These four IVIFSs cannot be distinguished using the measures S_1 to S_4. Two IVIFSs are seen to have identical measures if the interval means of μ and the interval means of ν are the same. For the measure S_5, $\alpha_4 > \alpha_1$ which is completely counter-intuitive. These functions are thus not efficient in differentiating IVIFSs. A new scoring function capable of discriminating between two IVIFS can be seen in Section 3.7.

3. Decision making framework

In this framework, a decision making takes place in an interval-valued intuitionistic fuzzy environment. With the input uncertainties and the human fuzzy processing, it is more advisable to have interval range values than an absolute value. In a scenario of multi-judge, multi-criteria decision making, it is improbable to reach an unanimous decision every single time. Each judge will have his/her own sense of judgment and the decision making may vary from one to another. Therefore, in this paper, independent assessments by each judge are considered for evaluation.

3.1. Decision maker's assessment

Let there be n alternatives $A_1, A_2, \ldots, A_n$ and q decision makers (DM) labeled as $DM_1, DM_2, \ldots, DM_q$ respectively. The decision makers assess each alternative based on m criteria $(C_1, C_2, \ldots, C_m)$ independently. During an assessment,both membership and non-membership values are assigned to the alternative for each criterion. The membership values are represented by $\mu = [\underline{\mu}, \overline{\mu}]$ where $\underline{\mu}$ is the lower boundary and $\overline{\mu}$ is the upper boundary. Similarly, the non-membership values are represented by $\nu = [\underline{\nu}, \overline{\nu}]$. $A_iC_j[\underline{\mu}, \overline{\mu}]$ indicates the degrees to which the alternative A_i satisfies the criterion C_j. $A_iC_j[\underline{\nu}, \overline{\nu}]$ indicates the degrees to which the alternative A_i does not satisfy the criterion C_j. The values for μ and ν can range from 0 to 1, adhering to the conditions of an IVIFS. Each criterion is judged independently of the other. With the given interval-valued membership and non-membership degrees, the IVIFS assessment table is formed for each decision-maker separately.

3.2. Selection approach

In practical situations, it is common that decision-makers have some degree of hesitation in making a decision. This indecisiveness is represented by the hesitancy factor $[\underline{h}, \overline{h}]$. Hesitation is an integral part of human decision making. Despite having hesitancy to a certain degree, decisions have to be made for actions to be taken. Decision results can vary depending on the ideology of the approach taken. A generalized fuzzy technique for order preference similarity to an ideal solution was proposed by Dwivedi et al. (2018) where a performance matrix from an IVIFS is obtained using $\mu = (1 - \lambda)\underline{\mu} + \lambda\overline{\mu}$ and $\nu = \lambda\underline{\nu} + (1 - \lambda)\overline{\nu}$. The method is sensitive to the preference parameter λ which is the degree of optimism but does not consider the influence

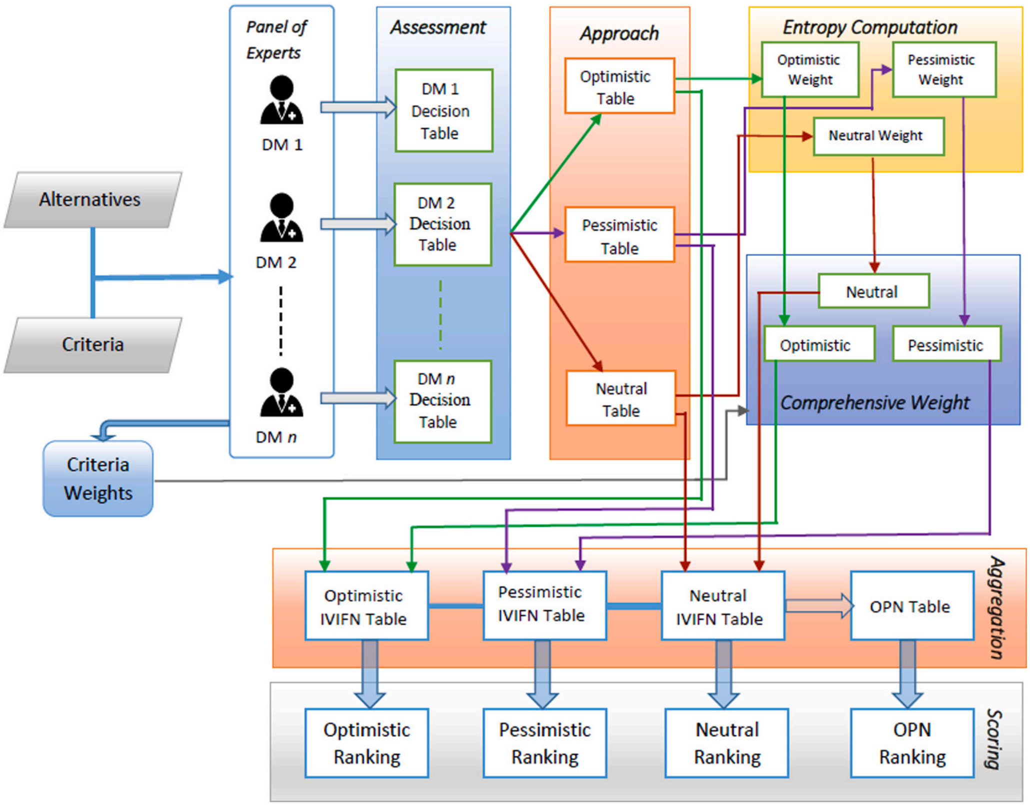

Fig. 1. The framework of decision making of the proposed system, from the evaluation of decision-makers (DM) on alternatives and criteria to the subject's ranking determination. The edges of green, purple and red colors represent the flow of information related to optimistic, pessimistic, and neutral ranking, respectively.

of hesitation. Yager (2002) considered optimism and pessimism as two extremes on one dimension continuum whereas other studies (Conway et al., 2008; Nicholls et al., 2008) suggested optimism and pessimism exist in divergent dimensions. Having one is not equivalent to the lack of another, membership and non-membership are not complementary. They are independent and should be dealt with separately when both are present simultaneously (Raufaste & Vautier, 2008).

3.2.1. Optimistic

In the optimistic approach, the notion of decision making is more inclined to the degree of membership. Having a membership degree, μ=1 is the most idealistic situation for optimism. We here estimate the confident membership by assuming ν=0 and completely diminishing the hesitancy factor associated with the evaluation. Eq. (15) gives the Confident Membership Estimate (CME):

$$CME = \frac{\underline{\mu} + \overline{\mu} - \underline{h} - \overline{h}}{2} \tag{15}$$

CME will help in constructing a table called Optimistic Table where the emphasis is given to the criterion's degree of memberships. With the available assessment tables, we estimate the confident membership for each decision-maker in assessing A_iC_j. Each criterion of every alternative is evaluated by q different decision-makers. For a given alternative and criteria, the selected membership and non-membership values for the Optimistic Table belong to the decision-maker with the highest confident membership estimate, i.e,

$$\max\left(CME_k(A_iC_j)\right) \quad \forall k \in \{1, 2, \ldots, q\}$$

If two or more decision-makers have equal confident membership estimates for A_iC_j, then the one having a greater $\overline{\mu}$ is chosen. In case of further deadlock, the membership and non-membership values selected for the Optimistic Table belong to the decision-maker having a higher $\underline{\mu}$.

3.2.2. Pessimistic

In the pessimistic approach, the idea of decision making is more inclined to the degree of non-membership. Having a non-membership degree, ν=1 is the most idealistic situation for pessimism. We here estimate the confident non-membership by assuming μ=0 and completely diminishing the hesitancy factor associated with the evaluation. Eq. (16) gives the Confident Non-membership Estimate (CNE):

$$CNE = \frac{\underline{\nu} + \overline{\nu} - \underline{h} - \overline{h}}{2} \tag{16}$$

CNE helps in constructing a table called Pessimistic Table where the emphasis is given to the criterion's degree of non-memberships. With the available assessment tables, we estimate the confident non-membership for each decision-maker in assessing A_iC_j. Each criterion of every alternative is evaluated by q different decision-makers. For a given alternative and criteria, the selected membership and non-membership values for the Pessimistic Table belong to the decision-maker with the highest confident non-membership estimate i.e,

$$\max\left(CNE_k(A_iC_j)\right) \quad \forall k \in \{1, 2, \ldots, q\}$$

If two or more decision-makers have equal confident non-membership estimates for A_iC_j, then the one having a greater $\overline{\nu}$ is chosen. In case of further deadlock, the membership and non-membership values selected for the Pessimistic Table belong to the decision-maker having a higher $\underline{\nu}$.

3.2.3. Neutral

In this approach, the notion of decision making is neutral. Emphasis is neither given to the degree of membership nor to the degree of non-membership. The Neutral Table is constructed by averaging using mean. Each criterion of every alternative is evaluated by q different decision-makers. For a given alternative and criteria, the membership and non-membership values for the Neutral Table are obtained using:

$$\underline{\mu}_{ij} = \frac{\sum_{k=1}^{q} A_i C_j(\underline{\mu})_k}{q} \qquad \overline{\mu}_{ij} = \frac{\sum_{k=1}^{q} A_i C_j(\overline{\mu})_k}{q} \tag{17}$$

$$\underline{\nu}_{ij} = \frac{\sum_{k=1}^{q} A_i C_j(\underline{\nu})_k}{q} \qquad \overline{\nu}_{ij} = \frac{\sum_{k=1}^{q} A_i C_j(\overline{\nu})_k}{q} \tag{18}$$

For all the approach tables, the hesitancy factor can be obtained using Eq. (3) and Eq. (4).

3.3. Entropy

Entropy indicates the intrinsic order of a criterion. In this paper, we extend the entropy measure proposed by Zhang et al. (2019) to an interval-valued intuitionistic fuzzy environment.

For criterion j and alternative set i ($i = 1, 2, \dots, n$), entropy E is:

$$E_j = \frac{1}{n}\sum_{i=1}^{n}\cos\Big(\big(\frac{\underline{\mu}_{ij} + \overline{\mu}_{ij} - \overline{\nu}_{ij} - \underline{\nu}_{ij}}{2}\big)\big(1 - \frac{1}{2}(\frac{\underline{h}_{ij}^2 + \overline{h}_{ij}^2}{2})\big)\frac{\pi}{2}\Big) \tag{19}$$

Further calculation of entropy weight (β) for criteria set j ($j = 1, 2, \dots, m$) is done using:

$$\beta_j = \frac{1 - E_j}{\sum_{j=1}^{m}(1 - E_j)} \tag{20}$$

Each chosen approach will have its own entropy weights for the criteria. The higher the entropy of a criterion, the lesser the entropy weight associated with it.

3.4. Comprehensive weight

Before assessing the alternatives, a decision-maker collectively assign weightage to each criterion (α_1, α_2, $\cdots$, α_m). The assigned weights are subjective to decision-makers whereas the obtained entropy weights (β_1, β_2, $\cdots$, β_m) hold to the objective of the selected approach in decision making.

Comprehensive weight (w) for criteria j is computed using:

$$w_j = \frac{\alpha_j \beta_j}{\sum_{j=1}^{m}\alpha_j \beta_j} \tag{21}$$

This integrates the criterion subjectivity and objectivity into the decision making process.

3.5. Aggregation

Intuitionistic fuzzy aggregation operators were introduced by Xu (2007) and soon after uncertain dynamic intuitionistic fuzzy weighted averaging (UDIFWA) operator was put forth by Xu and Yager (2008). Using the same, all the criteria of an alternative A_i are aggregated here.

$$A_i[C_1, C_2, \dots, C_m] = w_1 C_1 \oplus w_2 C_2 \oplus \cdots \oplus w_m C_m$$

This aggregation operation can be better rewritten as:

$$A_i(C_1, C_2, \dots, C_m) = \Big([1 - \prod_{j=1}^{m}(1 - \underline{\mu}_{ij})^{w_j}, 1 - \prod_{j=1}^{m}(1 - \overline{\mu}_{ij})^{w_j}], [\prod_{j=1}^{m}(\underline{\nu}_{ij})^{w_j}, \prod_{j=1}^{m}(\overline{\nu}_{ij})^{w_j}], [\prod_{j=1}^{m}(1 - \overline{\mu}_{ij})^{w_j} - \prod_{j=1}^{m}(\overline{\nu}_{ij})^{w_j}, \prod_{j=1}^{m}(1 - \underline{\mu}_{ij})^{w_j} - \prod_{j=1}^{m}(\underline{\nu}_{ij})^{w_j}]\Big) \tag{22}$$

where w_j is the comprehensive weight of respective criterion C_j, $j = \{1, 2, \dots, m\}$, $w_j \in$ [0,1] and $\sum_{j=1}^{m} w_j = 1$.

The selected approach table is weighted with its own respective comprehensive weights of the criteria. This aggregation operator helps to wholly evaluate an alternative with respect to each criterion. Zhou and Xu (2017) showed that the values aggregated by fuzzy weighted averaging operators are always greater than those obtained by fuzzy weighted geometric operators and also introduced extreme aggregation operators. Decision making with these operators involve a lot of parameter specifications and hence is not employed in the framework.

3.6. OPN approach

OPN approach associates itself with the three different ideologies of decision making, namely Optimistic, Pessimistic, and Neutral. A generalized formula for aggregating IVIFS tables attained by d distinctive ideologies is given as:

$$\begin{aligned} &\text{UDIFWA}(A_i(t_1), A_i(t_2), \dots, A_i(t_d)) \\ &= \Big([1 - \prod_{z=1}^{d}(1 - \underline{\mu}_{t_z i})^{1/d}, 1 - \prod_{z=1}^{d}(1 - \overline{\mu}_{t_z i})^{1/d}], \\ &[\prod_{z=1}^{d}(\underline{\nu}_{t_z i})^{1/d}, \prod_{z=1}^{d}(\overline{\nu}_{t_z i})^{1/d}], [\prod_{z=1}^{d}(1 - \overline{\mu}_{t_z i})^{1/d} \\ &- \prod_{z=1}^{d}(\overline{\nu}_{t_z i})^{1/d}, \prod_{z=1}^{d}(1 - \underline{\mu}_{t_z i})^{1/d} - \prod_{z=1}^{d}(\underline{\nu}_{t_z i})^{1/d}]\Big) \end{aligned} \tag{23}$$

where A_i is the alternative and t_z is the selected approach table. This assimilates the different mindsets into one.

3.7. Measuring function and ranking

Measure is an evaluation tool for interpreting and discerning elements. Here, we introduce a new measuring function called Measured Membership Score (MMS). The MMS values are calculated using the upper and lower bounds of the membership degrees and the hesitancy factors. It is a cumulative likelihood representation of something occurring. For a selected approach IVIFS table, MMS for an alternative is computed using:

$$MMS(A_i) = \min\big[\underline{\mu}_i(1 + \overline{h}_i),\ \overline{\mu}_i(1 + \underline{h}_i)\big] \tag{24}$$

This measuring score function is a global evaluation of each alternative. The alternatives are ranked based on their MMS. The greater the MMS is, the more preferred the alternative.

The properties of the proposed MMS function are as follows:

Property 1: If $\underline{\mu} = 0$ then MMS = 0

Property 2: If $\underline{\mu} = \overline{\mu} = 1$ then MMS = 1

Property 3: If $[\underline{h}, \overline{h}] = [0, 0]$ then MMS = $\underline{\mu}$

Property 4: For any IVIFN $\big([\underline{\mu}, \overline{\mu}], [\underline{\nu}, \overline{\nu}]\big)$, MMS $\in [0, 1]$

A diagrammatic description of the decision-making framework is given in Fig. 1.

4. Discussion and evaluation

IVIFSs are effective in tackling uncertainty and inconsistency occurring in the process of decision making. Given an IVIFS, it can be seen to belong inside the region of the triangle ONM (Fig. 2). It presents a geometrical representation of decision making in the IVIFS environment. M is the conceptually the most affirmative point as $u = 1$, $v = h = 0$. N is the most unfavorable point with $v = 1$, $u = h = 0$. Point O is the most indecisive point as $h = 1$ and $u = v = 0$. Line MN represents the IVIFS which is absolutely confident and has no scope of indecisiveness as $h = 0$. The closer the line parallel to MN, the higher the confidence. All points on a line paralleled to MN share the same hesitancy factor.

Each of the differently shaded region in Fig. 2 represents a region where an IVIFS would tend to lie when taking a different mindset approach for decision making. For the optimistic region, focus is on the membership degree having an increasing tendency where μ can range from $\underline{\mu}$ to 1, while ν is restricted to [0, $\underline{\nu}$]; whereas for the pessimistic region, focus is on the non-membership degree having an increasing tendency where ν can range from $\underline{\nu}$ to 1, while μ is restricted to [0, $\underline{\mu}$]. The mutually neutral region favor the upper limits of membership and non-membership degrees with the lower bound of hesitancy free to increase any of the two belief degrees. The triangle formed by the yellow dotted lines and line segment of MN represents an unbiased neutral region.

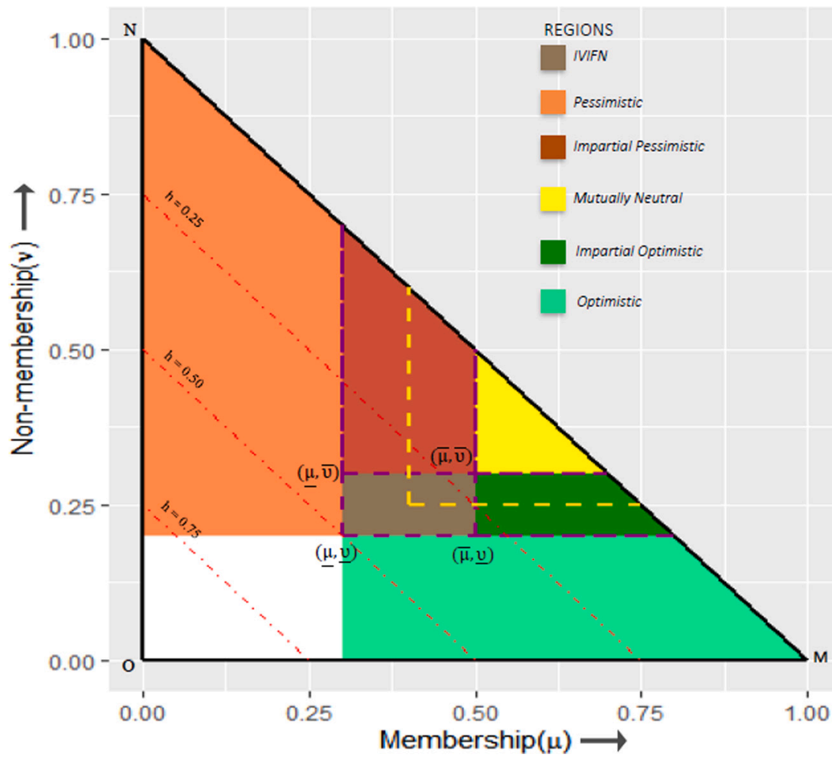

Fig. 2. Decision Making in Interval-valued Intuitionistic Fuzzy Environment. Here, $\underline{\mu}$ and $\overline{\mu}$ are lower and upper bound of membership degree respectively, whereas $\underline{\nu}$ and $\overline{\nu}$ are lower and upper bound of non-membership degree respectively.

The IVIFN region is shared by impartial optimistic region as well as the impartial pessimistic region. The impartial optimistic region holds the non-membership degree within its given range $[\underline{\nu}, \overline{\nu}]$, and membership degree is formed by adding the entire or portion of hesitancy factor to it. Similarly, the impartial pessimistic region holds the membership degree within its given range $[\underline{\mu}, \overline{\mu}]$ and the non-membership degree is formed by adding the entire or portion of hesitancy factor to it.

4.1. Extended entropy and its comparison

The smaller the area of the triangle formed by the IVIFS, the more conducive it is for estimation. This measure of area is taken into consideration while calculating the entropy of IVIFS. For a given IVIFS, $\underline{h}$ and $\overline{h}$ are the two extremes which will construct the occupied space. The extended entropy measure we proposed considers the hesitancy factor as well as employs the difference between membership and non-membership degrees, which is seen to be an integral part of most entropy methods. Using the two facilitates in capturing the fuzziness as well as the intuition in decision making. Concurrently, the cosine function helps in restraining the entropy measurements to [0,1]. When the difference between membership and non-membership degrees is equivalent for two IVIFSs, then the one with a higher hesitancy factor ends up having a greater entropy score.

Szmidt and Kacprzyk (2001) introduced entropy to intuitionistic fuzzy sets and laid out axiomatic properties it needs to fulfill. A function E is said to be an entropy measure of an IVIFS $A = \{(x_i, M_A(x_i), N_A(x_i)) \mid x_i \in X\}$, where $M_A(x_i) = [\underline{\mu}_A(x_i), \overline{\mu}_A(x_i)]$ and $N_A(x_i) = [\underline{\nu}_A(x_i), \overline{\nu}_A(x_i)]$ if it fulfills the following properties:

(1) $E(A) = 0$ iff A is a crisp set
(2) $E(A) = 1$ iff $M_A(x_i) = N_A(x_i)$
(3) $E(A) = E(A^c)$
(4) $E(A) \leq E(B)$, for $\forall x_i \in X$ iff $M_A(x_i) \leq M_B(x_i)$ and $N_A(x_i) \geq N_B(x_i)$ for $M_B(x_i) \leq N_B(x_i)$ or $M_A(x_i) \geq M_B(x_i)$ and $N_A(x_i) \leq N_B(x_i)$ for $M_B(x_i) \geq N_B(x_i)$

The extended entropy formula, Eq. (19) satisfies all the above conditions of an intuitionistic fuzzy environment. Table 1 shows a comparison between existing entropy measures and the one we proposed, E_{NEW}. This comparison demonstrate that E_{NEW} is able to differentiate given IVIFSs while the existing entropy measures may not. For a chosen entropy measure, the IVIFSs where the distinction has not been made are highlighted in a similar color. For a given case A([0.4,0.5],[0.4,0.5]) where membership degree is the same as non-membership degree, $E_{JY}(A) = E_{WC}(A) = E_{LJ}(A) = E_{WZ}(A) = E_{NEW}(A) = 1.00$ in accordance to the above-defined axiomatic properties of entropy measure whereas $E_{ZX}(A) = 0.90$ as it is based on distance-based definitions.

These entropy measures provided the reliability of information but is not a rational choice for ranking of IVIFNs. To say the least, an IVIFN and its complement will bear the same entropy value and hence cannot be differentiated.

4.2. New measuring function and its comparison

Chen et al. (2012) proposed a ranking method to overcome the drawback of the accuracy function of Ye (2009), and later (Wu & Chiclana, 2014) presented a risk attitudinal ranking method for IVIFNs based on novel score and accuracy expected functions. Garg (2016a) introduced an improved score function for IVIFSs incorporating a weighted hesitancy factor:

$$GIS(\alpha) = \frac{\underline{\mu} + \overline{\mu}}{2} + k_1 \underline{\mu}(1 - \underline{\mu} - \underline{\nu}) + k_2 \overline{\mu}(1 - \overline{\mu} - \overline{\nu}) \quad (25)$$

Interval-valued Pythagorean fuzzy environment was explored by Garg (2016b) and a novel accuracy function for multiple-criteria decision-making (MCDM) was proposed. This method takes into account the degree of hesitation as in Eq. (26) :

$$M(\alpha) = \frac{\underline{\mu}^2 - \sqrt{1 - \underline{\mu}^2 - \underline{\nu}^2} + \overline{\mu}^2 - \sqrt{1 - \overline{\mu}^2 - \overline{\nu}^2}}{2} \quad (26)$$

$$D = \begin{array}{c} A1 \\ A2 \\ A3 \\ A4 \end{array} \begin{bmatrix} ([0.4,0.5],[0.3,0.4]) & ([0.4,0.6],[0.2,0.4]) & ([0.1,0.3],[0.5,0.6]) & ([0.2,0.4],[0.4,0.5]) \\ ([0.6,0.7],[0.2,0.3]) & ([0.6,0.7],[0.2,0.3]) & ([0.4,0.7],[0.1,0.2]) & ([0.4,0.6],[0.3,0.4]) \\ ([0.3,0.6],[0.3,0.4]) & ([0.5,0.6],[0.3,0.4]) & ([0.5,0.6],[0.1,0.3]) & ([0.3,0.5],[0.4,0.5]) \\ ([0.7,0.8],[0.1,0.2]) & ([0.6,0.7],[0.1,0.3]) & ([0.3,0.4],[0.1,0.2]) & ([0.1,0.3],[0.5,0.6]) \end{bmatrix}$$

Box I.

Table 1
Entropy measure comparison. In this table, A1 to A7 represents 7 cases of IVIFN, whereas E_{JY}, E_{WC}, E_{LJ}, E_{ZX}, E_{WZ} and E_{NEW} represent six different entropy measures, i.e. Jun Ye's Entropy (Ye, 2010), Wei et al. entropy (Wei et al., 2011), Liu Jing's entropy (Jing, 2013), Zhang, Xing et al. entropy (Zhang et al., 2014), Wei and Zhang's entropy (Wei & Zhang, 2015) and our proposed entropy.

IVIFN	E_{JY}	E_{WC}	E_{LJ}	E_{ZX}	E_{WZ}	E_{NEW}
A1([0.1,0.2],[0.3,0.4])	0.96	0.76	0.87	0.50	0.98	0.96
A2([0.4,0.5],[0.4,0.5])	1.00	1.00	1.00	0.90	1.00	1.00
A3([0.5,0.5],[0.0,0.0])	0.74	0.50	0.67	0.50	0.87	0.77
A4([0.5,0.5],[0.1,0.1])	0.83	0.56	0.71	0.60	0.90	0.83
A5([0.5,0.5],[0.3,0.3])	0.96	0.71	0.83	0.80	0.97	0.95
A6([0.6,0.6],[0.2,0.2])	0.83	0.50	0.67	0.60	0.87	0.81
A7([0.7,0.7],[0.2,0.2])	0.74	0.38	0.55	0.50	0.76	0.71

Here, we showcase the proposed framework and measuring function for decision making with a practical example from a real-world scenario. Comparison is made with some of the approaches as well as validation is done for the reliability and effectiveness of the method.

Example 1: An investor wants to make an investment and has four alternatives to choose from: Car company (A1); Food company (A2); Computer company (A3); Arms company (A4). The decision must be based on four major criteria:(C1) High growth; (C2) Low Risk; (C3) Social impact; (C4) Environment impact. The possible alternatives are assessed w.r.t all the mentioned criteria in an interval-valued intuitionistic fuzzy environment by an expert. The assessment is provided in the form of a decision matrix D given as in Box I.

The weights given to C1, C2, C3, C4 are 0.15, 0.25, 0.35 and 0.25 respectively. The proposed approach is utilized to find the best alternative. Here the decision-making is done by a single expert, therefore the optimistic, pessimistic, and neutral approaches represent the same IVIFS table as obtained by the decision maker's assessment. Each alternative is aggregated, scored, and ranked. The ranking obtained is $A2 > A4 > A3 > A1$ making $A2$, computer company the best alternative for investment.

Table 2 provides a ranking comparison overview of different approaches with various measuring functions for the above-mentioned decision making problem. The best and the worst cases obtained by the proposed approach concurs with the result of other approaches.

4.3. Validation of the framework

To evaluate the validity of the MCDM method and consistency in the ranking, Wang and Triantaphyllou (2008) established three criteria:

Criteria 1 - *The method should not alter the ranking index of the best alternative when one of the remaining alternatives is replaced with a worse alternative.* Suppose a non-optimal alternative A_i is replaced by another alternative A_i' which is less optimal than A_i and the ranking of alternatives is done again, then the index of the best alternative should not change.

Criteria 2 - *Transitivity property must hold good for the ranking of the alternatives.* Assuming the MCDM problem is randomly broken down into smaller problems consisting of two alternatives each, then the ranking obtained must satisfy the transitivity property. Say, if $A_2 > A_1$ and $A_1 > A_3$ then $A_2 > A_3$ must hold.

Criteria 3 - *If the decision-making problem is decomposed into smaller sub-problems and the MCDM method is used for ranking the alternatives then the combined ranking of the sub-problems should be identical to the ranking of the original un-decomposed problem.*

The proposed method is checked for the three criteria and is seen to be validated and have consistency in the ranking.

Checking Criteria 1: The non-optimal alternative $A1$ is replaced with a worse alternative $A1'$ having ([0.2,0.3], [0.4,0.5]) assigned to $C1$ and ([0.3,0.5], [0.2,0.4]), ([0.1,0.2], [0.6,0.7]), ([0.3,0.4], [0.3,0.5]) assigned to $C2$, $C3$, $C4$ respectively. With the proposed method, the evaluated value of A1 tends to 0.3122. The new ranking order of the alternatives becomes $A2 > A4 > A3 > A1'$. This new ranking is similar to that with the previous and $A2$ remains the best alternative in both cases.

Checking Criteria 2 and 3: Supposing, the problem is decomposed into smaller sub problems $\{A1, A2, A3\}$, $\{A1, A2, A4\}$ and $\{A2, A3, A4\}$ and the proposed approach is applied, the obtained respective rankings are $\{A2 > A3 > A1\}$, $\{A2 > A4 > A1\}$, $\{A2 > A4 > A3\}$. When the obtained rankings are combined, it results in $A2 > A4 > A3 > A1$ which is same as the original ranking.

Example 1. We present another example and compare the results obtained from the framework with those from other existing approaches. In this example, a real cloud computing service problem as used in Düğenci (2016) is adapted. Four potential cloud services providers are : SAP Sales on Demand (A1), Salesforce Sales Cloud (A2), Microsoft Dynamic CRM (A3), and Oracle Cloud CRM (A4). These cloud service providers need to be evaluated for final selection. Four experts evaluated their services on five attributes namely performance(C1), payment(C2), reputation(C3), scalability(C4), and security(C5). The expert ratings on the services can be found in Düğenci (2016).

Table 3 gives a comparison for the service alternatives ranking obtained through various approaches. It can be clearly seen that the ranking given by the proposed method is in line with the other methods except (Zhang et al., 2014), which has a reverse order of ranking as it assumes the middle point ([0.5, 0.5], [0.5, 0.5]) to be the equilibrium. The proposed score and framework is therefore seen to produce a stable and reliable ranking of alternatives for decision making. The framework is also computationally more efficient than a model of linear programming (Chen, 2016) where computation increases exponentially with sample size.

5. Ranking of risky gait

5.1. Participants

Thirty young, healthy college-goers (23 male and 7 female; age of 24 ± 2 years) took part in this study. Each of them was familiar with technology and asserted to using mobile phone while walking, on an everyday basis. Written consent was provided by each participant and procedure approved by the ethics committee (IIRG004B-19HWB).

5.2. Protocol

Qualisys motion capture system with six high-speed Oqus cameras was set up across a 10 m long and 2 m wide walkway. Reflective

Table 2
Ranking: MMS vs Other scoring functions. A1, A2, A3 and A4 are four cases presented in Example 1.

	Ranking	Evaluated value for the alternatives			
		A1	A2	A3	A4
Chen et al. (2012)	A2 ≻ A4 ≻ A3 ≻ A1	0.1002	0.1734	0.1443	0.1636
Garg (2016a)	A2 ≻ A3 ≻ A4 ≻ A1	0.3795	0.6615	0.5895	0.5260
Garg (2016b)	A2 ≻ A3 ≻ A4 ≻ A1	−0.6763	−0.4115	−0.5350	−0.5398
Ye (2009)	A2 ≻ A3 ≻ A4 ≻ A1	0.0627	0.3879	0.3106	0.0691
Proposed method	A2 ≻ A4 ≻ A3 ≻ A1	0.3585	0.6551	0.5791	0.6096

Table 3
Ranking comparison with existing approaches. In this table, A1, A2, A3 and A4 represent the four cloud services discussed in Example 1.

	Service alternatives ranking
Düğenci (2016)	A3 ≻ A1 ≻ A2 ≻ A4
Xu and Yager (2008)	A3 ≻ A1 ≻ A2 ≻ A4
Zhang et al. (2014)	A4 ≻ A2 ≻ A1 ≻ A3
Proposed method	A3 ≻ A1 ≻ A2 ≻ A4

markers were attached to the subject's hip, knee, heel, and toe of both legs to capture their movement. Each subject walked across the designated walkway ten times at their own comfortable pace. Subjects were then required to use mobile phones and answer a set of questions while walking. They were free to use the internet to find the answers. This engaged their attention, leading to distraction from the primary task of walking. The movements of the lower extremity were considered for motion analysis. Gait cycles of the subjects were extracted from the continuous motion.

5.3. Data

650 complete gait cycles for each of the two attempts were analyzed. A gait cycle starts with the heel strike of the right or left foot and ends with the subsequent heel strike of the same foot. Temporal–spatial features considered to be the most fundamental and reliable gait features by practitioners and geriatrics alike (Preiningerova et al., 2015) were used for gait analysis. Jhawar et al. (2016) used instance selection for better identification of distorted gaits. Here, we focus on observational gait characteristics, namely, stride length, step width, and cadence. Stride length is the distance between two footsteps of the same foot. The lateral separating distance between the centers of the two-foot in the double limb part during the gait cycle is called step width. Cadence is the rate at which a person moves, expressed in the number of steps taken in a minute. Fig. 3 gives a visual representation of the considered spatial gait features. Six out of the thirty participants were found having distracted gaits. The medical experts classified them as people with risky gaits and being more prone to falling, and selected them to benefit from support intervention. It would not be a problem for the experts to attend six patients and monitor their progress. But in the case of a larger number of patients with risky gaits waiting to get support intervention, it will get cumbersome and very difficult for the medical experts to handle and prioritize them. To address this issue, a grading system for risky gaits comes in handy to ease the situation. It will help the doctors to attend the patients in the order of their risk of falling.

5.4. Assessment

For risky gaits, which is a multi-attribute decision making problem, let $P = \{P_1, P_2, \ldots, P_n\}$ be a set of n patients and A be a set of m attributes $(A_1, A_2, \ldots, A_m)$. A number of q decision-makers make their evaluations separately and are not influenced by others. For each patient, all the attributes are evaluated independently and the assessment is given in the form of membership and non-membership intervals. Using the proposed framework, we rank the risky patients for support care. The decision-makers providing their assessments on the gaits are rehabilitation physicians. Here, we have three decision-makers

Table 4
Decision maker 1 assessment table : $P_i, i = \{1, 2, \ldots, 6\}$ represent the six patients taking the experiment, whereas A_1, A_2 and A_3 represent three attributes being recorded. μ is the membership degree that a patient has the attribute, and ν is the membership degree that a patient does not have the attribute.

	A_1		A_2		A_3	
	μ	ν	μ	ν	μ	ν
P_1	0.5–0.6	0.1–0.2	0.5–0.6	0.1–0.2	0.5–0.7	0.0–0.1
P_2	0.5–0.6	0.3–0.4	0.3–0.6	0.2–0.4	0.5–0.7	0.1–0.2
P_3	0.7–0.9	0.0–0.1	0.8–0.9	0.1–0.1	0.7–0.8	0.0–0.1
P_4	0.3–0.4	0.4–0.5	0.4–0.5	0.4–0.5	0.4–0.6	0.3–0.4
P_5	0.7–0.8	0.1–0.2	0.7–0.8	0.1–0.2	0.6–0.7	0.1–0.2
P_6	0.6–0.8	0.0–0.2	0.7–0.8	0.1–0.2	0.4–0.6	0.1–0.3

Table 5
Decision maker 2 assessment table : $P_i, i = \{1, 2, \ldots, 6\}$ represent the six patients taking the experiment, whereas A_1, A_2 and A_3 represent three attributes being recorded. μ is the membership degree that a patient has the attribute, and ν is the membership degree that a patient does not have the attribute.

	A_1		A_2		A_3	
	μ	ν	μ	ν	μ	ν
P_1	0.4–0.6	0.1–0.3	0.4–0.5	0.1–0.2	0.4–0.7	0.2–0.3
P_2	0.4–0.6	0.3–0.4	0.5–0.6	0.2–0.4	0.5–0.6	0.2–0.3
P_3	0.7–0.8	0.0–0.1	0.8–0.9	0.0–0.1	0.8–0.9	0.0–0.1
P_4	0.4–0.5	0.4–0.5	0.2–0.4	0.5–0.6	0.5–0.6	0.3–0.4
P_5	0.6–0.7	0.1–0.2	0.5–0.7	0.1–0.3	0.5–0.6	0.2–0.4
P_6	0.6–0.8	0.1–0.2	0.4–0.6	0.1–0.3	0.5–0.7	0.2–0.3

Table 6
Decision maker 3 assessment table : $P_i, i = \{1, 2, \ldots, 6\}$ represent the six patients taking the experiment, whereas A_1, A_2 and A_3 represent three attributes being recorded. μ is the membership degree that a patient has the attribute, and ν is the membership degree that a patient does not have the attribute.

	A_1		A_2		A_3	
	μ	ν	μ	ν	μ	ν
P_1	0.3–0.4	0.2–0.3	0.35–0.65	0.10–0.25	0.35–0.55	0.15–0.35
P_2	0.55–0.65	0.25–0.35	0.35–0.55	0.3–0.4	0.55–0.65	0.10–0.25
P_3	0.60–0.75	0.10–0.25	0.7–0.8	0.0–0.1	0.75–0.85	0.0–0.1
P_4	0.35–0.45	0.35–0.45	0.4–0.6	0.3–0.4	0.4–0.6	0.1–0.4
P_5	0.5–0.7	0.1–0.2	0.65–0.75	0.1–0.2	0.6–0.7	0.1–0.2
P_6	0.6–0.7	0.0–0.2	0.5–0.6	0.1–0.2	0.5–0.6	0.1–0.2

Table 7
Confident membership estimates are computed using Eq. (15) after decision makers DM_1, DM_2 and DM_3 reviewed the attributes A_1, A_2 and A_3 demonstrated on patients P_1 to P_6.

	A_1			A_2			A_3		
	DM_1	DM_2	DM_3	DM_1	DM_2	DM_3	DM_1	DM_2	DM_3
P_1	0.25	0.20	−0.05	0.25	0.05	0.175	0.25	0.35	0.15
P_2	0.45	0.35	0.50	0.20	0.40	0.25	0.35	0.35	0.375
P_3	0.65	0.55	0.525	0.80	0.75	0.55	0.55	0.75	0.65
P_4	0.15	0.35	0.20	0.35	0.15	0.35	0.35	0.45	0.25
P_5	0.65	0.45	0.35	0.65	0.40	0.55	0.45	0.40	0.45
P_6	0.50	0.55	0.40	0.65	0.20	0.25	0.20	0.45	0.25

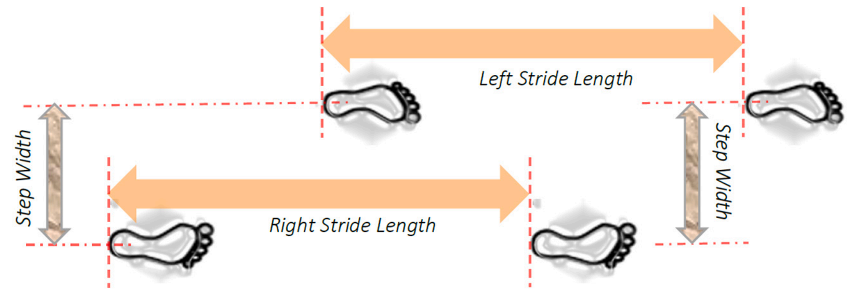

Fig. 3. Gait spatial characteristics.

Table 8
Optimistic table. Three attributes, i.e. A_1, A_2 and A_3 of patients, P_1 to P_6 are evaluated. For each patient and each attribute, the upper and lower bound of membership degrees ($\overline{\mu}$ and $\underline{\mu}$), the upper and lower bound of non-membership degrees ($\overline{\nu}$ and $\underline{\nu}$), and the upper and lower bound of hesitancy factors ($\overline{h}$ and $\underline{h}$) are presented.

	A_1						A_2						A_3					
	$\underline{\mu}$	$\overline{\mu}$	$\underline{\nu}$	$\overline{\nu}$	$\underline{h}$	$\overline{h}$	$\underline{\mu}$	$\overline{\mu}$	$\underline{\nu}$	$\overline{\nu}$	$\underline{h}$	$\overline{h}$	$\underline{\mu}$	$\overline{\mu}$	$\underline{\nu}$	$\overline{\nu}$	$\underline{h}$	$\overline{h}$
P_1	0.5	0.6	0.1	0.2	0.2	0.4	0.5	0.6	0.1	0.2	0.2	0.4	0.4	0.7	0.2	0.3	0.0	0.4
P_2	0.55	0.65	0.25	0.35	0.0	0.2	0.5	0.6	0.2	0.4	0.0	0.3	0.55	0.65	0.1	0.25	0.1	0.35
P_3	0.7	0.9	0.0	0.1	0.0	0.3	0.8	0.9	0.1	0.1	0.0	0.1	0.8	0.9	0.0	0.1	0.0	0.2
P_4	0.4	0.5	0.4	0.5	0.0	0.2	0.4	0.6	0.3	0.4	0.0	0.3	0.5	0.6	0.3	0.4	0.0	0.2
P_5	0.7	0.8	0.1	0.2	0.0	0.2	0.7	0.8	0.1	0.2	0.0	0.2	0.6	0.7	0.1	0.2	0.1	0.3
P_6	0.6	0.8	0.1	0.2	0.0	0.3	0.7	0.8	0.1	0.2	0.0	0.2	0.5	0.7	0.2	0.3	0.0	0.3

who assessed six patients based on three attributes, i.e. stride length (A_1), step width (A_2) and cadence for their chances of falling (A_3). The weights assigned to the attributes are (0.4,0.3,0.3) respectively. The result of assessments are shown in Tables 4–6. The higher the membership degree for an attribute, the more it indicates support for the likelihood of falls in a given patient.

5.5. Approaches

5.5.1. Optimistic approach

In this approach, the focus is on an attribute's membership degrees which represents the likelihood of a patient being risky faller. With Eq. (15), we calculate and present the CME of gait attributes for all the three decision-makers in Table 7. These CME values help in constructing the Optimistic Table. For a given patient and attribute, the selected membership and non-membership values for Optimistic Table (Table 8) belong to the decision-maker with highest CME, or membership degree in case of a deadlock.

To illustrate, let us take the case of Patient 1 Attribute 1. The CME of three decision-makers for the same are 0.25, 0.20 and −0.05 respectively. It is observed that DM_1 has the highest CME. The IVIFS selected in this case for the optimistic table belongs to DM_1. Therefore, ([0.5,0.6],[0.1,0.2]) represents ($[\underline{\mu}, \overline{\mu}]$,$[\underline{\nu}, \overline{\nu}]$).

Based on the Optimistic Table (Table 8), we calculate entropy weights β for the attributes which capture the objectivity of the approach. Using Eq. (19) and Eq. (20), we get $\beta_1 = 0.339$, $\beta_2 = 0.356$ and $\beta_3 = 0.305$. Comprehensive weights w for the attributes are then obtained by Eq. (21). The obtained weights $w_1 = 0.41$, $w_2 = 0.32$ and $w_3 = 0.27$ are coupled with their respective attributes in the Optimistic Table. Aggregation operations are performed utilizing Eq. (22) for each patient as to have a collective weighted representation for ranking. The aggregated IVIFS for optimistic approach is seen in Table 9.

5.5.2. Pessimistic approach

In this approach, the focus is on an attribute's non-membership degrees which represents the likelihood of a patient not being risky faller. With Eq. (16), we calculate and present the CNE of gait attributes for all the three decision-makers in Table 10. These CNE values help in constructing the Pessimistic Table. For a given patient and attribute, the selected membership and non-membership values for Pessimistic

Table 9
Aggregated IVIFS for optimistic table. In this table, the upper and lower bound of membership degrees ($\overline{\mu}$ and $\underline{\mu}$), the upper and lower bound of non-membership degrees ($\overline{\nu}$ and $\underline{\nu}$), and the upper and lower bound of hesitancy factors ($\overline{h}$ and $\underline{h}$) of each patient (P_1 to P_6) are aggregated.

	$\underline{\mu}$	$\overline{\mu}$	$\underline{\nu}$	$\overline{\nu}$	$\underline{h}$	$\overline{h}$
P_1	0.475	0.63	0.121	0.223	0.147	0.405
P_2	0.535	0.635	0.182	0.334	0.032	0.284
P_3	0.764	0.9	0.0	0.1	0.0	0.236
P_4	0.429	0.562	0.338	0.438	0.0	0.234
P_5	0.676	0.777	0.1	0.2	0.023	0.224
P_6	0.613	0.777	0.121	0.223	0.0	0.267

Table 10
Confident non-membership estimates are computed using Eq. (15) after decision makers DM_1, DM_2 and DM_3 reviewed the attributes A_1, A_2 and A_3 demonstrated on patients P_1 to P_6.

	A_1			A_2			A_3		
	DM_1	DM_2	DM_3	DM_1	DM_2	DM_3	DM_1	DM_2	DM_3
P_1	−0.15	−0.10	−0.15	−0.15	−0.25	−0.15	−0.30	0.05	−0.05
P_2	0.25	0.20	0.20	0.05	0.15	0.15	−0.10	0.05	−0.05
P_3	−0.10	−0.15	0.025	0.05	−0.05	−0.15	−0.15	−0.05	−0.10
P_4	0.25	0.35	0.20	0.35	0.40	0.20	0.20	0.25	0.0
P_5	0.05	−0.05	−0.10	0.05	0.0	0.0	−0.05	0.15	−0.05
P_6	−0.10	0.0	−0.15	0.05	−0.10	−0.15	−0.10	0.10	−0.15

Table (Table 11) belong to the decision-maker with highest CNE, or non-membership degree in case of a deadlock.

To illustrate, let us take the case of Patient 2 Attribute 2. The CNE of three decision-makers for the same are 0.05, 0.15 and 0.15 respectively. It is observed that DM_2 and DM_3 have obtained equal CNE value. Upon checking $\overline{\nu}$ for Patient 2 Attribute 2 in Tables 4–6, we found that both decision-makers have $\overline{\nu} = 0.4$. Therefore, the selection is done based on higher $\underline{\nu}$. The IVIFS selected in this case for the Pessimistic Table belongs to DM3, as $\underline{\nu} = 0.2$ for DM2 and $\underline{\nu} = 0.3$ for DM3. Accordingly, ([0.35,0.55],[0.3,0.4]) represents ($[\underline{\mu}, \overline{\mu}]$,$[\underline{\nu}, \overline{\nu}]$).

Based on the Pessimistic Table (Table 11), we calculate entropy weights β for the attributes which capture the objectivity of the approach. Using Eq. (19) and Eq. (20), we get $\beta_1 = 0.296$, $\beta_2 = 0.412$ and $\beta_3 = 0.292$. Comprehensive weights w for the attributes are then obtained by Eq. (21). The obtained weights $w_1 = 0.36$, $w_2 = 0.37$ and

Table 11
Pessimistic table. Three attributes, i.e. A_1, A_2 and A_3 of patients, P_1 to P_6 are evaluated. For each patient and each attribute, the upper and lower bound of membership degrees ($\overline{\mu}$ and $\underline{\mu}$), the upper and lower bound of non-membership degrees ($\overline{\nu}$ and $\underline{\nu}$), and the upper and lower bound of hesitancy factors ($\overline{h}$ and $\underline{h}$) are presented.

	A1						A2						A3					
	$\underline{\mu}$	$\overline{\mu}$	$\underline{\nu}$	$\overline{\nu}$	$\underline{h}$	$\overline{h}$	$\underline{\mu}$	$\overline{\mu}$	$\underline{\nu}$	$\overline{\nu}$	$\underline{h}$	$\overline{h}$	$\underline{\mu}$	$\overline{\mu}$	$\underline{\nu}$	$\overline{\nu}$	$\underline{h}$	$\overline{h}$
P_1	0.4	0.6	0.1	0.3	0.1	0.5	0.35	0.65	0.1	0.25	0.1	0.55	0.4	0.7	0.2	0.3	0.0	0.4
P_2	0.5	0.6	0.3	0.4	0.0	0.2	0.35	0.55	0.3	0.4	0.05	0.35	0.5	0.6	0.2	0.3	0.1	0.3
P_3	0.6	0.75	0.1	0.25	0.0	0.3	0.8	0.9	0.1	0.1	0.0	0.1	0.8	0.9	0.0	0.1	0.0	0.2
P_4	0.4	0.5	0.4	0.5	0.0	0.2	0.2	0.4	0.5	0.6	0.0	0.3	0.5	0.6	0.3	0.4	0.0	0.2
P_5	0.7	0.8	0.1	0.2	0.0	0.2	0.7	0.8	0.1	0.2	0.0	0.2	0.5	0.6	0.2	0.4	0.0	0.3
P_6	0.6	0.8	0.1	0.2	0.0	0.3	0.7	0.8	0.1	0.2	0.0	0.2	0.5	0.7	0.2	0.3	0.0	0.3

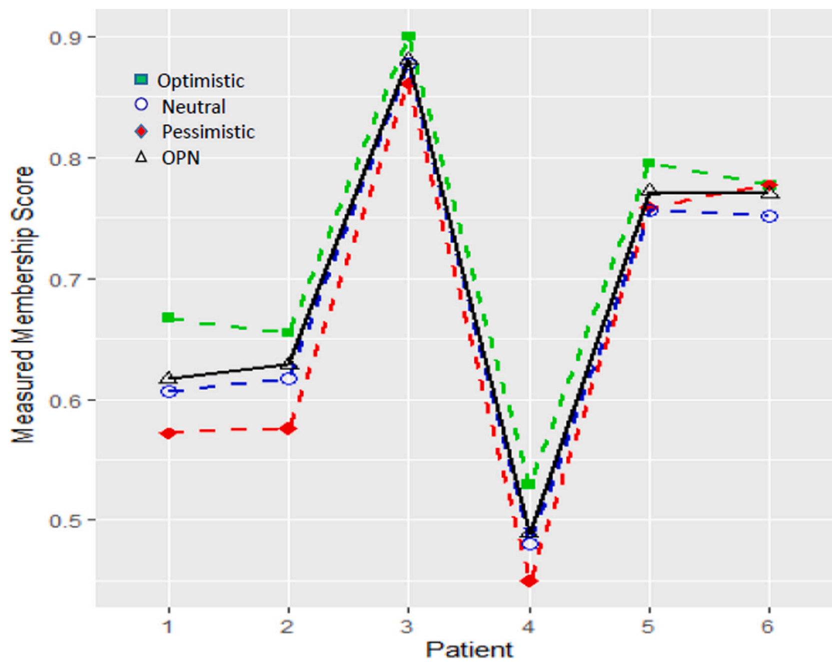

Fig. 4. Score Comparison for Different Approaches.

Table 12
Aggregated IVIFS for pessimistic table. In this table, the upper and lower bound of membership degrees ($\overline{\mu}$ and $\underline{\mu}$), the upper and lower bound of non-membership degrees ($\overline{\nu}$ and $\underline{\nu}$), and the upper and lower bound of hesitancy factors ($\overline{h}$ and $\underline{h}$) of each patient (P_1 to P_6) are aggregated.

	$\underline{\mu}$	$\overline{\mu}$	$\underline{\nu}$	$\overline{\nu}$	$\underline{h}$	$\overline{h}$
P_1	0.382	0.648	0.121	0.28	0.072	0.497
P_2	0.449	0.582	0.269	0.37	0.048	0.282
P_3	0.743	0.861	0.0	0.139	0.0	0.257
P_4	0.365	0.496	0.402	0.504	0.0	0.233
P_5	0.656	0.759	0.121	0.241	0.0	0.224
P_6	0.618	0.777	0.121	0.223	0.0	0.261

$w_3 = 0.27$ are coupled with their respective attributes in the Pessimistic Table. Aggregation operations are performed utilizing Eq. (22) for each patient as to have a collective weighted representation for ranking. The aggregated IVIFS for pessimistic approach is seen in Table 12.

5.5.3. Neutral approach

In this approach, an attribute's membership and non-membership degrees are treated equally. Confidence estimations of the decision-makers' assessments are inessential. The Neutral Table is constructed by averaging the assessments provided by decision-makers. For a given patient and criterion, the membership and non-membership values for the Neutral Table as shown in Table 13 are calculated using Eq. (17) and eq(18).

Based on the Neutral Table (Table 13), we calculate entropy weights β for the attributes which capture the objectivity of the approach. Using Eq. (19) and Eq. (20), we get $\beta_1 = 0.326$, $\beta_2 = 0.335$ and $\beta_3 = 0.339$. Comprehensive weights w for the attributes are then obtained by Eq. (21). The obtained weights $w_1 = 0.39$, $w_2 = 0.30$ and $w_3 = 0.31$ are coupled with their respective attributes in the Neutral Table. Aggregation operations are performed utilizing Eq. (22) for each patient as to have a collective weighted representation for ranking. The aggregated IVIFS for neutral approach is seen in Table 14.

5.5.4. OPN approach

This approach is a fusion of Optimistic, Pessimistic, and Neutral Approaches. These three approaches are aggregated using Eq. (23) where each approach is given equal an weightage. The aggregated IVIFS table for the OPN approach is shown in Table 15.

5.6. Scores and ranking

For each of the approaches, patients are evaluated based on their aggregated IVIFSs by the new MMS function. The membership degrees affirms the importance of belief. For a decision to be made, the hesitancy in totality needs to be considered. Membership degrees accumulated with hesitancy extremes for different approaches are shown in Table 16. The minimum of two membership degrees, which acts as the new MMS are summarized in Table 17, and graphical comparison is highlighted in Fig. 4.

The patient with the highest MMS is at the greatest risk of falling. The rankings of the patients under different approaches are presented

Table 13
Neutral table. Three attributes, i.e. A_1, A_2 and A_3 of patients, P_1 to P_6 are evaluated. For each patient and each attribute, the upper and lower bound of membership degrees ($\overline{\mu}$ and $\underline{\mu}$), the upper and lower bound of non-membership degrees ($\overline{\nu}$ and $\underline{\nu}$), and the upper and lower bound of hesitancy factors ($\overline{h}$ and $\underline{h}$) are presented.

	A1						A2						A3					
	$\underline{\mu}$	$\overline{\mu}$	$\underline{\nu}$	$\overline{\nu}$	$\underline{h}$	$\overline{h}$	$\underline{\mu}$	$\overline{\mu}$	$\underline{\nu}$	$\overline{\nu}$	$\underline{h}$	$\overline{h}$	$\underline{\mu}$	$\overline{\mu}$	$\underline{\nu}$	$\overline{\nu}$	$\underline{h}$	$\overline{h}$
P_1	0.4	0.53	0.13	0.27	0.2	0.47	0.42	0.58	0.1	0.22	0.2	0.48	0.42	0.65	0.12	0.25	0.1	0.46
P_2	0.48	0.62	0.28	0.38	0.0	0.24	0.38	0.58	0.23	0.4	0.02	0.39	0.52	0.65	0.13	0.25	0.1	0.35
P_3	0.67	0.82	0.03	0.15	0.03	0.3	0.77	0.87	0.03	0.1	0.03	0.2	0.75	0.85	0.0	0.1	0.05	0.25
P_4	0.35	0.45	0.38	0.48	0.07	0.27	0.33	0.5	0.4	0.5	0.0	0.27	0.43	0.6	0.23	0.4	0.0	0.34
P_5	0.6	0.73	0.1	0.2	0.07	0.3	0.62	0.75	0.1	0.23	0.02	0.28	0.57	0.67	0.13	0.27	0.06	0.3
P_6	0.6	0.77	0.03	0.2	0.03	0.37	0.53	0.67	0.1	0.23	0.1	0.37	0.47	0.63	0.13	0.27	0.1	0.4

Table 14
Aggregated IVIFS for neutral table. In this table, the upper and lower bound of membership degrees ($\overline{\mu}$ and $\underline{\mu}$), the upper and lower bound of non-membership degrees ($\overline{\nu}$ and $\underline{\nu}$), and the upper and lower bound of hesitancy factors ($\overline{h}$ and $\underline{h}$) of each patient (P_1 to P_6) are aggregated.

	$\underline{\mu}$	$\overline{\mu}$	$\underline{\nu}$	$\overline{\nu}$	$\underline{h}$	$\overline{h}$
P_1	0.412	0.585	0.117	0.248	0.167	0.471
P_2	0.465	0.618	0.208	0.339	0.043	0.327
P_3	0.728	0.846	0	0.117	0.037	0.272
P_4	0.37	0.516	0.33	0.459	0.025	0.299
P_5	0.597	0.719	0.108	0.229	0.052	0.294
P_6	0.542	0.703	0.068	0.229	0.068	0.39

Table 15
OPN IVIFS table. The findings in Tables 9, 12 and 14 are aggregated using Eq. (23) for each patient (P_1 to P_6). Here, the upper and lower bound of membership degrees ($\overline{\mu}$ and $\underline{\mu}$), the upper and lower bound of non-membership degrees ($\overline{\nu}$ and $\underline{\nu}$), and the upper and lower bound of hesitancy factors ($\overline{h}$ and $\underline{h}$).

	$\underline{\mu}$	$\overline{\mu}$	$\underline{\nu}$	$\overline{\nu}$	$\underline{h}$	$\overline{h}$
P_1	0.424	0.622	0.12	0.249	0.129	0.456
P_2	0.484	0.612	0.217	0.347	0.04	0.299
P_3	0.745	0.871	0.0	0.118	0.011	0.255
P_4	0.389	0.525	0.355	0.466	0.008	0.256
P_5	0.645	0.753	0.109	0.223	0.025	0.246
P_6	0.592	0.755	0.1	0.225	0.02	0.308

Table 16
Accumulated membership is computed for each approaches. $\overline{\mu}$ and $\underline{\mu}$ represent upper and lower bound of membership degrees respectively, and $\overline{h}$ and $\underline{h}$ represent the upper and lower bound of hesitancy factors respectively.

	Optimistic		Pessimistic		Neutral		OPN	
	$\underline{\mu}(1+\overline{h})$	$\overline{\mu}(1+\underline{h})$	$\underline{\mu}(1+\overline{h})$	$\overline{\mu}(1+\underline{h})$	$\underline{\mu}(1+\overline{h})$	$\overline{\mu}(1+\underline{h})$	$\underline{\mu}(1+\overline{h})$	$\overline{\mu}(1+\underline{h})$
P_1	0.667	0.723	0.572	0.695	0.606	0.683	0.617	0.702
P_2	0.687	0.655	0.576	0.610	0.617	0.645	0.629	0.636
P_3	0.944	0.900	0.934	0.861	0.926	0.877	0.935	0.881
P_4	0.529	0.562	0.450	0.466	0.481	0.529	0.489	0.529
P_5	0.827	0.795	0.803	0.759	0.773	0.756	0.804	0.772
P_6	0.777	0.777	0.779	0.777	0.753	0.751	0.774	0.770

Table 17
Score obtained by different approaches for patients P_1 to P_6.

	Optimistic	Pessimistic	Neutral	OPN
P_1	0.667	0.572	0.606	0.617
P_2	0.655	0.576	0.617	0.629
P_3	0.9	0.861	0.877	0.881
P_4	0.529	0.45	0.481	0.489
P_5	0.795	0.759	0.756	0.772
P_6	0.777	0.777	0.751	0.77

in Table 18. It is observed that for all approaches, P3 is considered the patient with the highest risk of falling, and P4 has the lowest risk.

The patients can undergo rehabilitation according to their obtained ranking from the selected approach. Having a shortfall of experts in rehabilitation care to address a large population is a demanding situation. This sustainable ranking system will help in moderating and organizing support intervention for patients with risky gaits.

Table 18
Patient ranking by different approaches.

	Optimistic	Pessimistic	Neutral	OPN
Rank1	3	3	3	3
Rank2	5	6	5	5
Rank3	6	5	6	6
Rank4	1	2	2	2
Rank5	2	1	1	1
Rank6	4	4	4	4

6. Conclusion

The proposed approach driven system is capable to provide ranking of risky gaits according to different attitudes instead of a single optimal ranking. It captures the confidence of decision-makers in their assessments. The method takes into consideration μ, ν, h based on geometrical meaning. Furthermore, the entropy measure for IVIFSs is extended and a new measuring function. This method is computationally efficient and effective for healthcare decision making even when the alternative samples get large. The framework produces reliable results. However it requires complete assessment information from the decision makers and is not handy in case of missing data. Also, each criterion is assumed to be independent of each other and the method does not consider any interdependency between them. These limitations will be solved in the future work.

CRediT authorship contribution statement

Abhishek Jhawar: Experiment, Methodology, Writing. **Chee Kau Lim:** Methodology, Investigation, Validation. **Chee Seng Chan:** Conceptualization, Software, Resources.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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